Dispersion calculation for lower hybrid waves in the current sheet of reconnection with guide field

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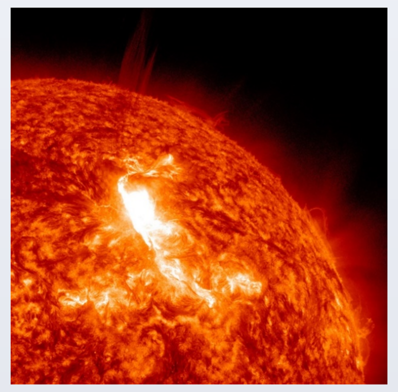
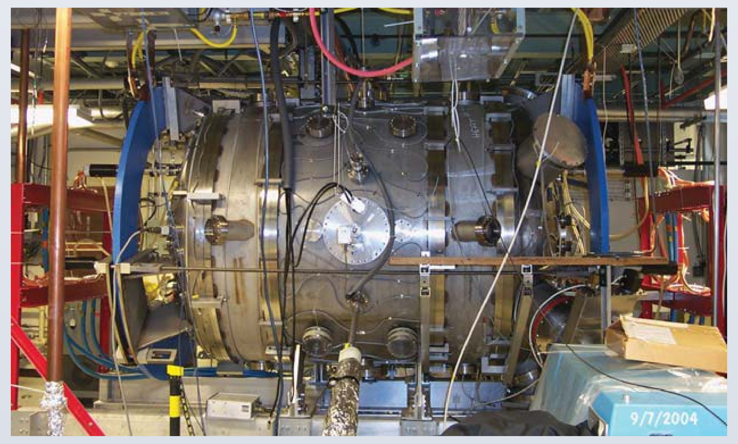
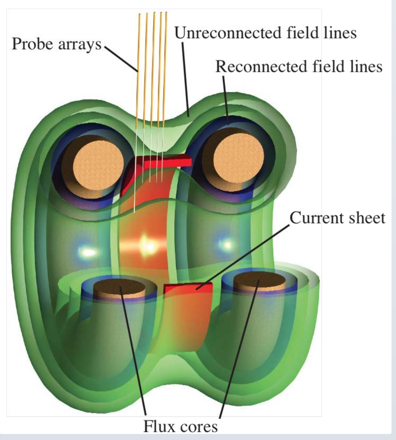
INTRODUCTION

Magnetic reconnection is the topological rearrangement of magnetic field lines resulting in the conversion of magnetic energy into particle kinetic and thermal energy. It plays an important role in phenomenon like solar flares and aurora.







The Magnetic Reconnection Experiment (MRX) studies the underlying physics of reconnection at the laboratory scale.

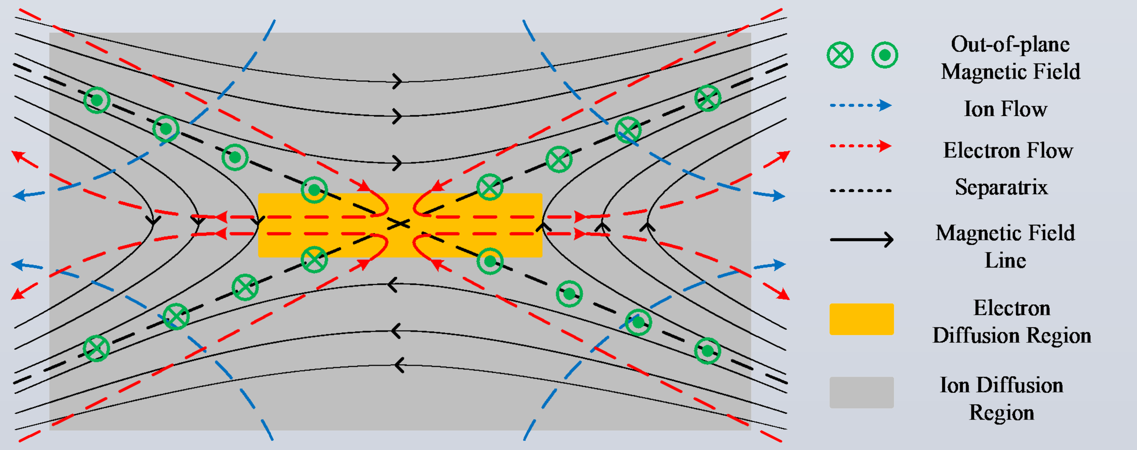


Figure 1. Two-fluid physics in the current sheet.

MOTIVATION

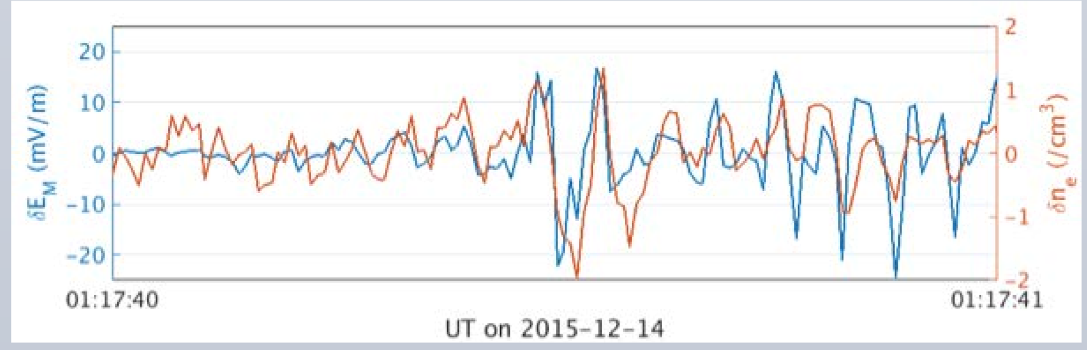


Figure 2. Electric field fluctuations and density fluctuations near the lower hybrid frequency correlate during reconnection events with guide field both in the laboratory and in space.

The anomalous resistivity generated by these lower hybrid-type waves may explain fast reconnection.

The dispersion and growth rates of obliquely propagating electromagnetic waves are calculated using local analysis to study why there are lower hybrid-type waves in the current sheet during reconnection with a guide field.

COLD LIMIT (ζ >> 1) EQUILIBRIUM AND LINEARIZATION

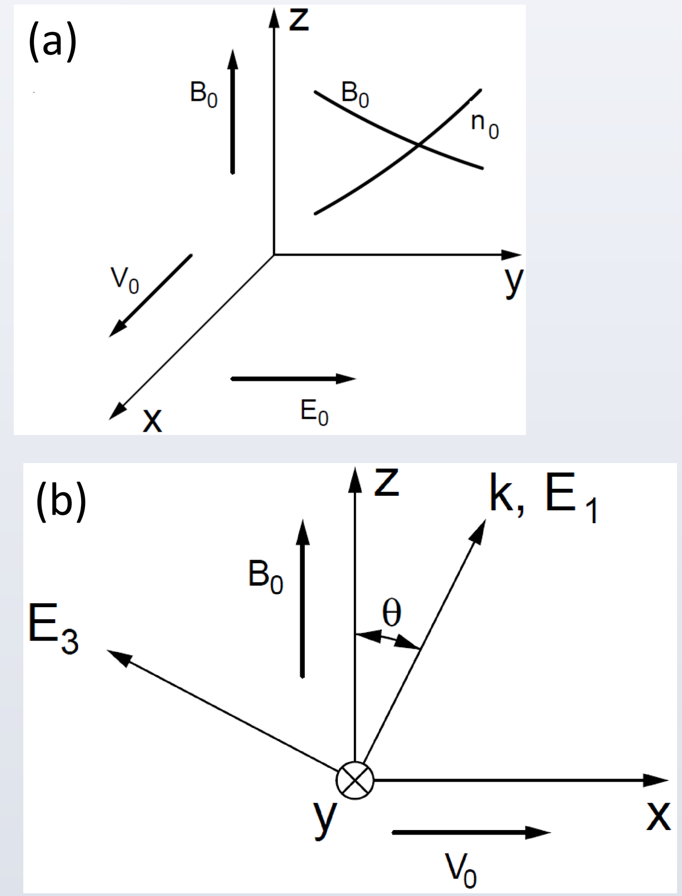


Figure 3. (a) Illustrations of the equilibrium state. Ions are at rest while electrons drift toward positive x direction, crossing magnetic field in the z direction. The resultant Lorentz force and electric field is balanced by pressure gradients in the y direction, which points towards the current sheet center. (b) Definitions of E_1 and E_3. E_2 is same as E_y.[1]

$$\zeta = x + iy = \frac{\omega}{kv_i} = \frac{\omega_{ci}c\Omega}{\omega_{pi}v_iK} = \frac{\alpha\Omega}{K}$$

ζ is the ratio of the phase velocity to the ion thermal velocity assuming a Maxwellian distribution function.[1]

$$n = i\frac{n_0 e}{M\omega^2}\left(\mathbf{k}\cdot\mathbf{E} - i\epsilon E_y\right) \approx i\frac{en_0}{M\omega^2}\left(\mathbf{k}\cdot\mathbf{E}\right)$$

$$\mathbf{j}^e \times \mathbf{B}_0 = en_0\mathbf{V}_0 \times \mathbf{B} + en_0\mathbf{E} + en\mathbf{E}_0$$

$$\mathbf{j}^i \approx i\frac{\omega_{pi}^2}{\omega}\epsilon_0\mathbf{E}$$

$$k_z^2E_x - k_xk_zE_z = i\omega\mu_0 j_x$$

$$k^2E_y = i\omega\mu_0 j_y$$

$$E_z + V_0B_y + ik_z\frac{T_e}{e}\frac{n}{n_0} = 0$$

$$\begin{pmatrix} D_{xx} & D_{xy} & D_{xz} \\ D_{yx} & D_{yy} & D_{yz} \\ D_{zx} & D_{zy} & D_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = 0,$$

$$D_{xx} = K^2\cos^2\theta + 1 - \frac{\beta_i}{\beta_e+\beta_i}\frac{KV\sin\theta}{\Omega}$$
$$D_{xy} = i(\Omega - KV\sin\theta)$$
$$D_{xz} = -K^2\sin\theta\cos\theta - \frac{\beta_i}{\beta_e+\beta_i}\frac{KV\cos\theta}{\Omega}$$
$$D_{yx} = -i\left(\Omega - \frac{\beta_e}{2}\frac{K^2\sin^2\theta}{\Omega}\right)$$

$$D_{yy} = K^2 + 1$$
$$D_{yz} = i\frac{\beta_e}{2}\frac{K^2\sin\theta\cos\theta}{\Omega}$$
$$D_{zx} = KV\cos\theta - \frac{\beta_e}{2}\frac{K^2\sin\theta\cos\theta}{\Omega}$$
$$D_{zy} = 0$$
$$D_{zz} = \Omega - KV\sin\theta - \frac{\beta_e}{2}\frac{K^2\cos^2\theta}{\Omega}.$$

$$\begin{aligned}&\Omega^4 - 2KV\sin\theta\Omega^3 \\ &- \left[\left(K^2+1\right)\left(K^2\cos^2\theta+1\right) - K^2V^2\sin^2\theta + \frac{\beta_e}{2}K^2\right]\Omega^2 \\ &+ KV\sin\theta\left[\beta_eK^2 + \left(K^2+1\right)\frac{\beta_e+2\beta_i}{\beta_e+\beta_i}\right]\Omega \\ &+ K^2\left[\frac{\beta_e}{2}\left[\left(K^2+1\right)^2\cos^2\theta - K^2V^2\sin^2\theta\right]\right. \\ &\left. -\left(K^2+1\right)V^2\frac{\beta_i}{\beta_e+\beta_i}\right] = 0.\end{aligned}$$

WARM REGIME (ζ ~ 1) EQUILIBRIUM AND LINEARIZATION

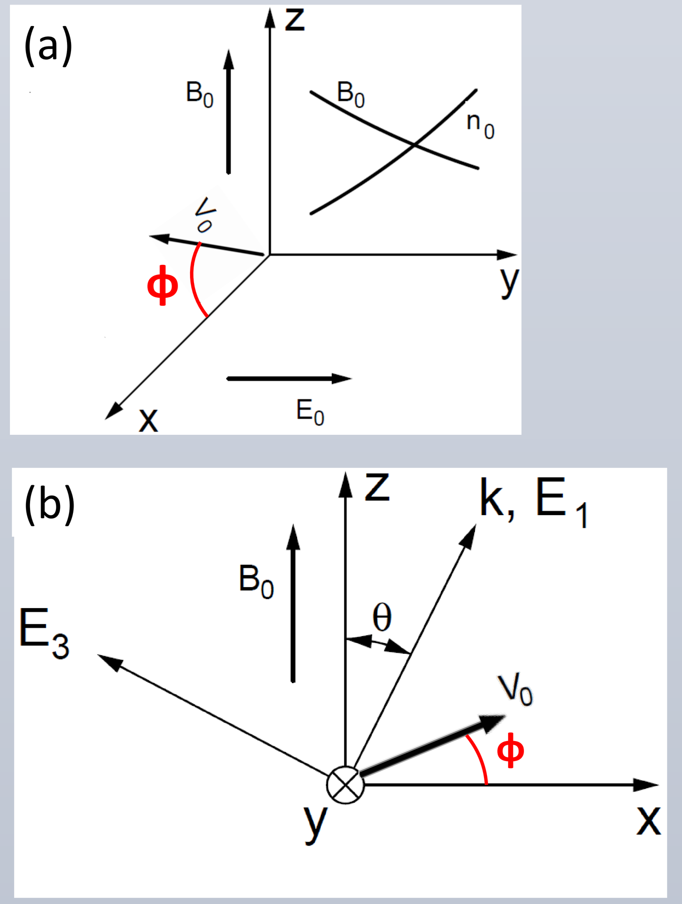


Figure 4. (a, b) Modified coordinate system for reconnection events with guide field.

Typical MRX plasma parameters
B_0 = 250 G
$T_i = T_e$ = 12 eV
n = 2.5 x 10^{13} cm^{-3}
V_0 = 2 x 10^7 cm/s
ϕ = 70º
θ = 70º

$$\zeta = x + iy = \frac{\omega}{kv_i} = \frac{\omega_{ci}c\Omega}{\omega_{pi}v_iK} = \frac{\alpha\Omega}{K}$$

$$Z \to -\frac{1}{\zeta} \text{ and } Z' \to \frac{1}{\zeta^2} \text{ when } \alpha \gg 1$$

$$n = i\frac{n_0 e}{Mk^2v_i^2}Z'(\zeta)\left(\mathbf{k}\cdot\mathbf{E} - \cancel{i\epsilon E_y}\right)$$

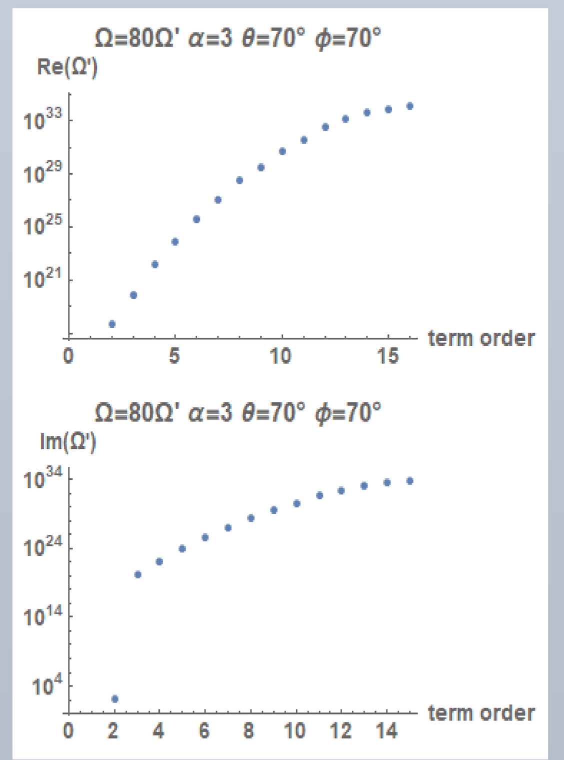
$$\mathbf{j}^e \times \mathbf{B}_0 = en_0\mathbf{V}_0 \times \mathbf{B} + en_0\mathbf{E} + en\mathbf{E}_0$$

$$\mathbf{j}^i = -i\frac{n_0e^2}{M}\frac{1}{kv_i}\left[Z(\zeta)\mathbf{E} - (\zeta Z' + Z)\left(\mathbf{E}\cdot\hat{\mathbf{k}}\right)\hat{\mathbf{k}} + \cancel{i(\epsilon/k)(\zeta Z' + Z)E_y\hat{\mathbf{k}}}\right]$$

$$k_z^2E_x - k_xk_zE_z = i\omega\mu_0 j_x$$

$$k^2E_y = i\omega\mu_0 j_y$$

$$E_z + V_0B_y + ik_z\frac{T_e}{e}\frac{n}{n_0} = 0$$



where

$$D_{xx} = K^2\cos^2(\theta) - \frac{VZ'\alpha\beta_i\zeta}{\beta_e+\beta_i}\sin(\theta)\cos(\phi) - \zeta\left(Z - \left(Z + Z'\zeta\right)\sin^2(\theta)\right)$$
$$D_{xy} = i\left(-KV\sin(\phi)\cos(\theta) - KV\sin(\theta)\cos(\phi) + \Omega\right)$$
$$D_{xz} = -K^2\sin(\theta)\cos(\theta) - \frac{VZ'\alpha\beta_i\zeta}{\beta_e+\beta_i}\cos(\phi)\cos(\theta) + \zeta\left(Z + Z'\zeta\right)\sin(\theta)\cos(\theta)$$
$$D_{yx} = i\left(KV\sin(\phi)\cos(\theta) + \frac{KZ'}{2}\alpha\beta_e\zeta\sin^2(\theta) - \Omega\right)$$
$$D_{yy} = K^2 - Z\zeta$$
$$D_{yz} = i\left(-KV\sin(\phi)\sin(\theta) + \frac{KZ'}{2}\alpha\beta_e\zeta\sin(\theta)\cos(\theta)\right)$$
$$D_{zx} = KV\cos(\phi)\cos(\theta) - \frac{\Omega Z'}{2}\alpha^2\beta_e\sin(\theta)\cos(\theta)$$
$$D_{zy} = 0$$
$$D_{zz} = -KV\sin(\theta)\cos(\phi) - \frac{\Omega Z'}{2}\alpha^2\beta_e\cos^2(\theta) + \Omega$$

Figure 5. The matrix equation yields a 15^{th} order polynomial in Ω, but it is physically justifiable to focus on the 5 highest order terms (as in the cold limit) and neglect lower order terms.

J-POLE APPROXIMATION

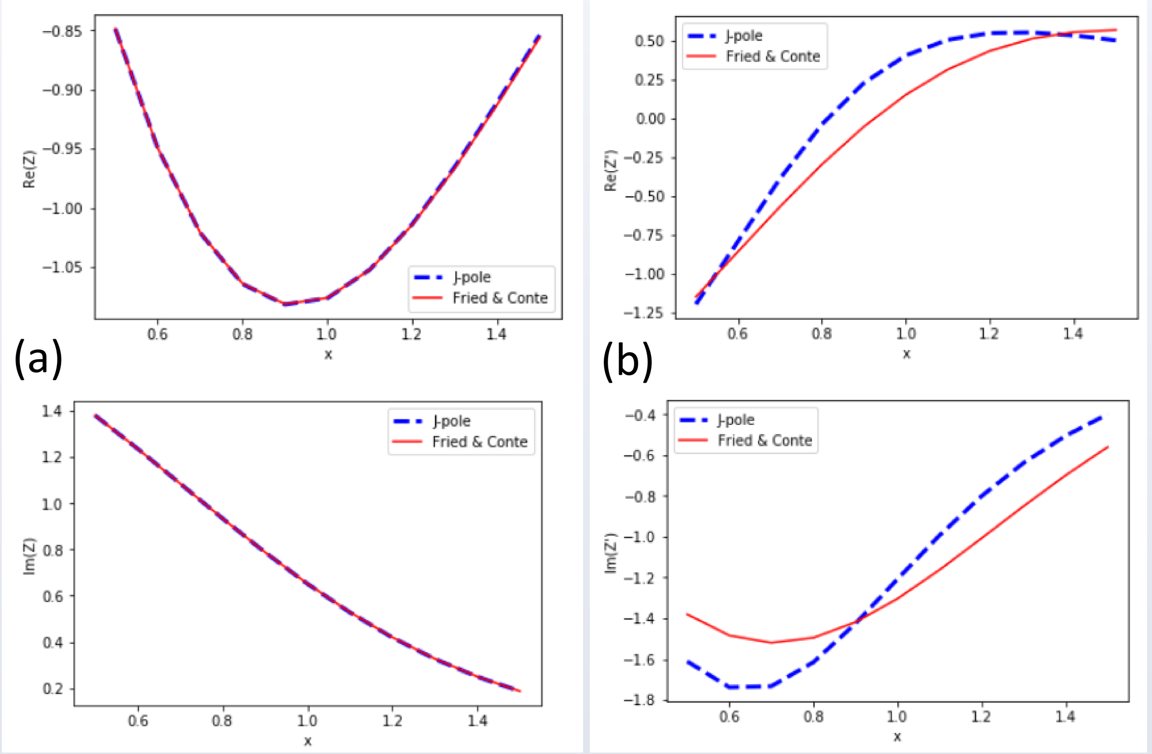


Figure 6. (a) 4-pole approximation for plasma dispersion function, Z.[2] (b) 2-pole approximation for Z'.[3] Here, ζ = x + iy ~ 1 (y = 0)

RESULTS

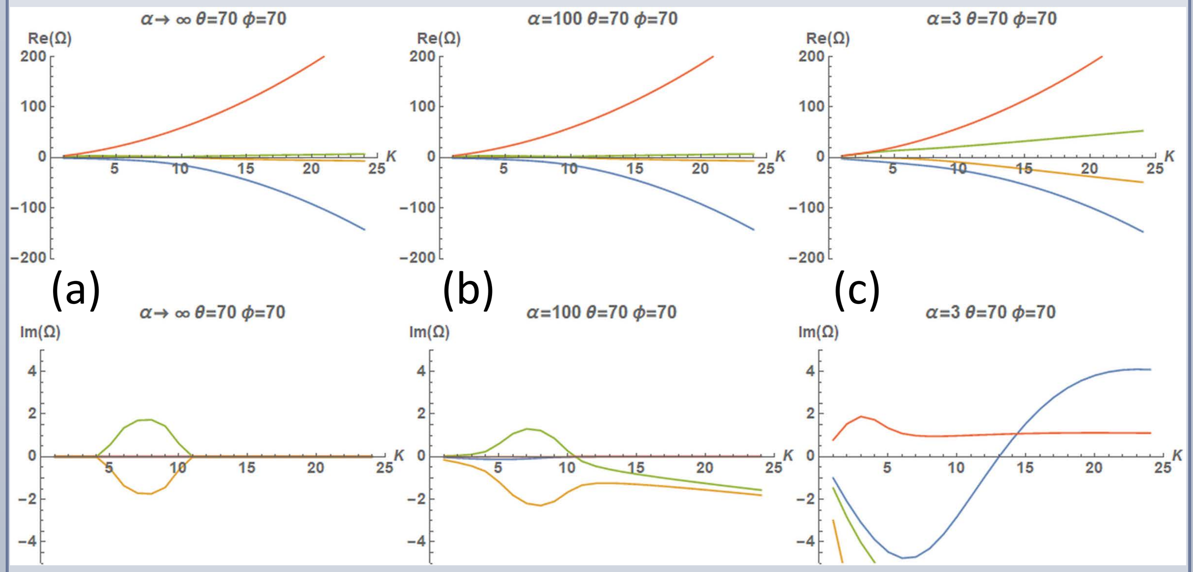


Figure 7. (top) Dispersion relation and (bottom) growth rates of wave modes during guide field reconnection when (a) α → ∞ (cold), (b) α = 100, and (c) α = 3 (warm), using typical MRX plasma parameters.

FUTURE WORK

1. **Try to solve the dispersion relation numerically without using the J-pole approximation**
2. **Study the correlation between the density and electric field fluctuations from these modes**
3. **Repeat the calculation for space parameters**

REFERENCES

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3. NRL Plasma Formulary. J. D. Huba. Naval Research Laboratory. (2016).

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